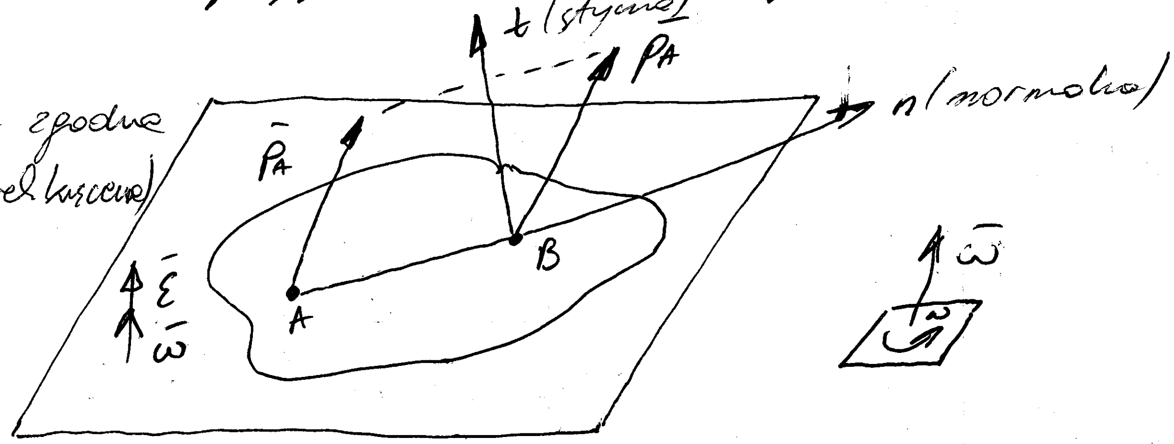
Zajęcia 2

Wyznaczanie przyspieszeń w ruchu płaskim

t - styczna zgodna z ε (kierunek kręcenia)



$$\bar p_B = \bar p_A + \bar\varepsilon \times \overline{AB} + \bar\omega \times (\bar\omega \times \bar\rho) \quad \langle 3D \rangle$$

$$\bar p_B = \bar p_A + \bar\varepsilon \times \overline{AB} - \omega^2 \overline{AB} \quad \langle 2D \rangle$$

$$p_{Bn} = p_A \cdot \cos\sphericalangle(p_A, n) - \omega^2 AB$$

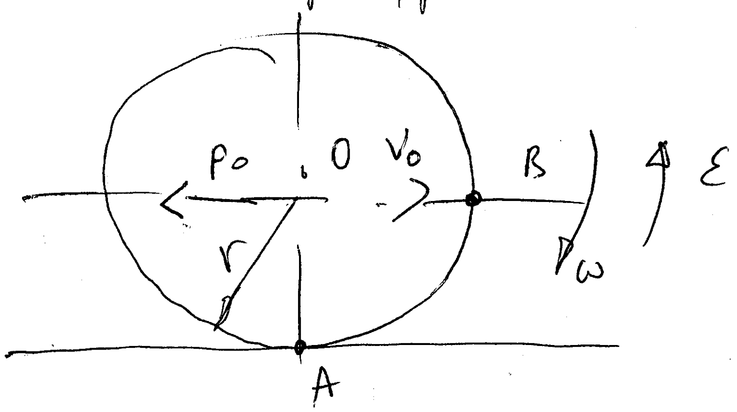
$$p_{Bt} = p_A \cos\sphericalangle(p_A, t) + \varepsilon AB$$

Zadanie 1

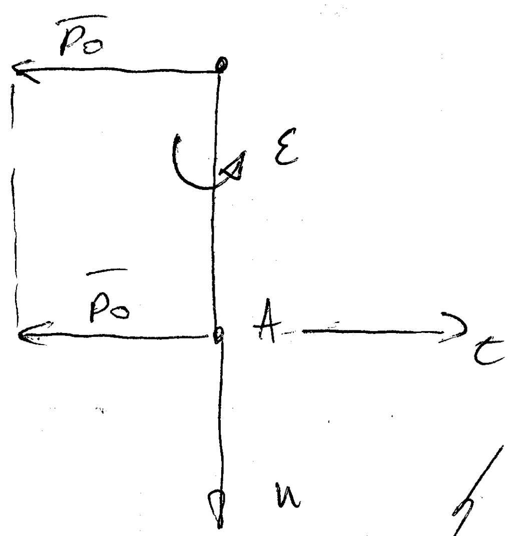
Oblicz przyspieszenie $\bar p_A$ na podstawie $\bar p_0$

Dane: r, V_0, p_0

Szukane: $\bar p_A$, $\bar p_B$



$$p_{An} = p_0 \cos \measuredangle(\bar{p_0}, \bar{n}) - \omega^2 OA$$
$$p_{At} = p_0 \cos \measuredangle(\bar{p_0}, t) + \varepsilon\, OA$$



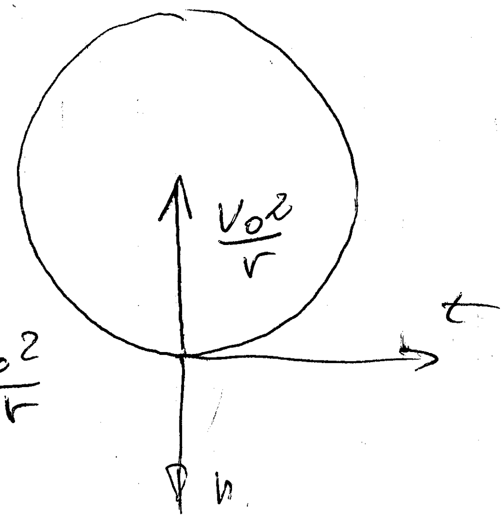
$$p_{An} = p_0 \cos 90^\circ - \omega^2 r$$
$$p_{At} = p_0 \cos 180^\circ + \varepsilon \cdot r$$
$$p_{An} = p_0 \cdot 0 - \omega^2 r$$
$$p_{At} = p_0 (-1) + \varepsilon r$$

Dla koła toczącego się bez poślizgu

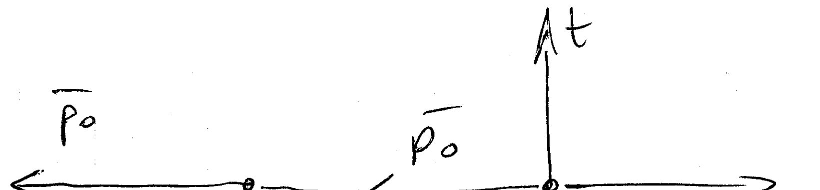
$$\begin{cases} v_0 = \omega r \\ p_0 = \varepsilon r \end{cases} \Rightarrow \quad \omega = \frac{v_0}{r} \quad \varepsilon = \frac{p_0}{r}$$

$$p_{An} = -\left(\frac{v_0}{r}\right)^2 \cdot r$$
$$p_{At} = -p_0 + \frac{p_0}{r} \cdot r$$
$$p_{An} = -\frac{v_0^2}{r}$$
$$p_{At} = -p_0 + p_0 = 0$$

$$p_A = \frac{v_0^2}{r}$$



Obliczamy przyspieszenie p_B na podstawie $\bar{p_0}$



$$p_{Bn} = p_0 \cdot \cos \measuredangle(\bar{p_0}, \bar{n}) - + \omega^2 OB$$
$$p_{Bt} = p_0 \cdot \cos \measuredangle(\bar{p_0}, \bar{t}) + + \varepsilon\, OB$$
$$p_{Bn} = p_0 \cdot \cos 180^\circ - \omega^2 r$$
$$p_{Bt} = p_0 \cos 90^\circ - \varepsilon r$$

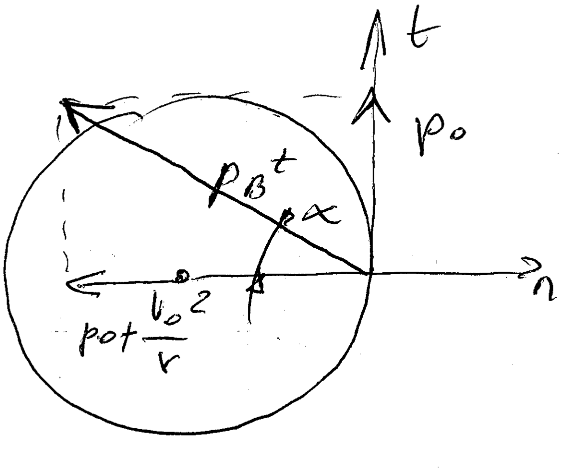
$p_{Bn} = -p_0 - \frac{V_0^2}{r}$ $\qquad p_{Bn} = -\left(p_0 + \frac{V_0^2}{r}\right)$

$p_{Bt} = p_0$

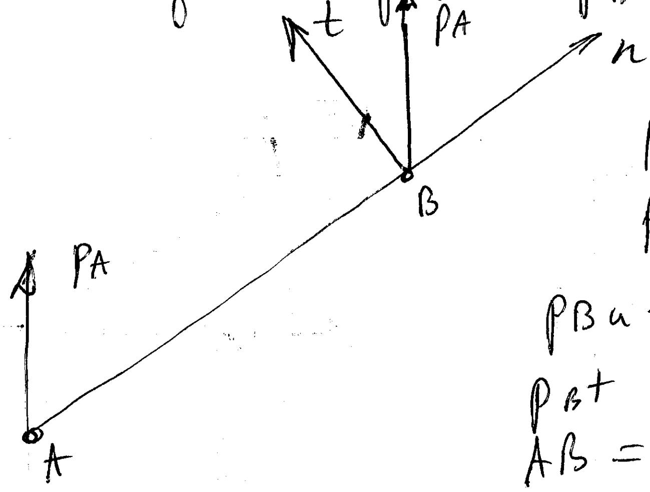
$p_B = \sqrt{p_{Bn}^2 + p_{Bt}^2}$

$p_B = \sqrt{\left(p_0 + \frac{V_0^2}{r}\right)^2 + p_0^2}$

$tg\alpha = \frac{p_0}{p_0 + \frac{V_0^2}{r}}$



Obliczamy raz jeszcze $\bar{p}_B$ na podstawie $\bar{p}_A$.



$p_{Bn} = p_A \cos\sphericalangle(\bar{p}_A, \bar{n}) - \omega^2 AB$

$p_{Bt} = p_A \cos\sphericalangle(\bar{p}_A, \bar{t}) + \varepsilon AB$

$p_{Bn} = p_A \cdot \cos 45^\circ - \omega^2 AB$

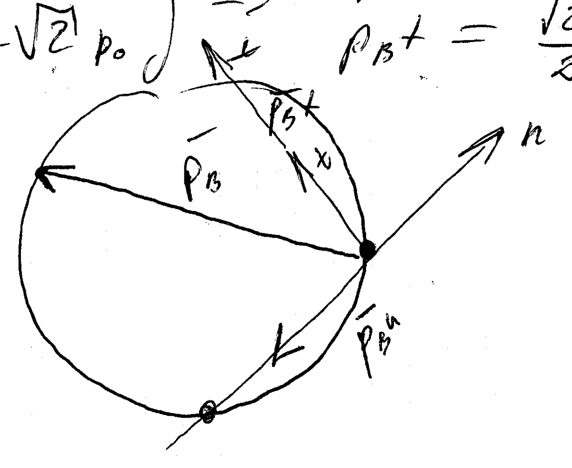
$p_{Bt} = p_A \cos 45^\circ + \varepsilon AB$

$AB = \sqrt{2} \cdot r$

$p_{Bn} = \frac{V_0^2}{r} \cdot \frac{\sqrt{2}}{2} - \frac{V_0^2}{r^2} \cdot \sqrt{2} \cdot r$

$p_{Bt} = \frac{V_0^2}{r} \cdot \frac{\sqrt{2}}{2} + \frac{p_0}{r} \cdot \sqrt{2} r$

$$\left.\begin{aligned} p_{Bn} &= \frac{V_0^2}{r}\left(\frac{\sqrt{2}}{2} - \sqrt{2}\right) \\ p_{Bt} &= \frac{\sqrt{2}}{2} \cdot \frac{V_0^2}{r} + \sqrt{2} p_0 \end{aligned}\right\} \Rightarrow \begin{aligned} p_{Bn} &= -\frac{\sqrt{2}}{2} \frac{V_0^2}{r} \\ p_{Bt} &= \frac{\sqrt{2}}{2} \frac{V_0}{r} + \sqrt{2} p_0 \end{aligned}$$



Dla sprawdzenia obliczmy moduły p_B z dwóch przypadków

moduł p_B

$$p_{B(0)}^2 = \left(p_0 + \frac{v_0^2}{r}\right)^2 + p_0^2$$

$$p_{B(A)}^2 = \left(-\frac{\sqrt{2}}{2}\frac{v_0^2}{r}\right)^2 + \left(\frac{\sqrt{2}}{2}\frac{v_0^2}{r} + \sqrt{2}\,p_0\right)^2$$

$$p_{B(0)}^2 = p_0^2 + 2p_0\frac{v_0^2}{r} + \frac{v_0^4}{r^2} + p_0^2$$

$$p_{B(A)}^2 = \frac{v_0^4}{2r^2} + \frac{v_0^4}{2r^2} + 2\frac{\sqrt{2}}{2}\cdot\frac{v_0^2}{r}\sqrt{2}\,p_0 + 2p_0^2$$

$$p_{B(0)}^2 = 2p_0^2 + 2p_0\frac{v_0^2}{r} + \frac{v_0^4}{r^2}$$

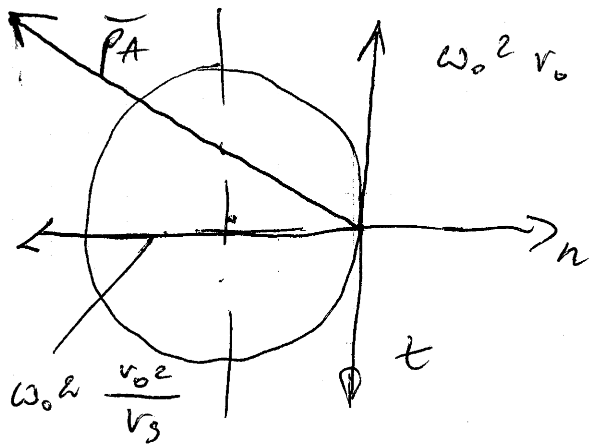
$$p_{B(A)}^2 = \frac{v_0^4}{r^2} + 2\frac{v_0^2}{r}p_0 + 2p_0^2$$

$$p_{An} = -\omega_0^2 r_0 \frac{r_0}{r_s}$$

$$p_{At} = -\omega_0^2 r_s$$

$$tg\,\alpha = \frac{r_s}{r_0} \qquad r_0 = \frac{r_2 + r_1}{2}$$

$$tg\,\alpha = \frac{r_2 - r_1}{r_1 + r_2} \qquad r_s = \frac{r_2 - r_1}{2}$$



$$\frac{r_0}{r_s} = \frac{r_1 + r_2}{r_2 - r_1} > 1$$

moduł p_A

$$|p_A|^2 = p_A^2 = p_{An}^2 + p_{At}^2 = \left(-\omega_0^2 r_0 \frac{r_0}{r_s}\right)^2 + \left(-\omega_0^2 r_s\right)^2 =$$

$$= \omega_0^4 r_0^2\left(\frac{r_0^2}{r_s^2} + 1\right) = \omega_0^4 r_0^2 \frac{r_0^2 + r_s^2}{r_s^2} =$$

$$= \omega_0^4\left(\frac{r_0}{r_s}\right)^2\left(r_0^2 + r_s^2\right) = \omega_0^4\left(\frac{r_1 + r_2}{r_2 - r_1}\right)^2\cdot$$

$$\cdot\left[\left(\frac{r_1 + r_2}{2}\right)^2 + \left(\frac{r_2 - r_1}{2}\right)^2\right] =$$

4

$$= \omega_0^4\left(\frac{r_1 + r_2}{r_2 - r_1}\right)^2\cdot\frac{1}{4}\left(r_1^2 + \cancel{2r_1 r_2} + r_2^2 + r_2^2 - \cancel{2r_1 r_2} + r_1^2\right) =$$

$$= \omega_0^4 \left(\frac{r_1+r_2}{r_2-r_1}\right)^2 \cdot \frac{2r_1^2+2r_2^2}{4} = \omega_0^4 \left(\frac{r_1+r_2}{r_2-r_1}\right)^2 \cdot \frac{r_1^2+r_2^2}{2}$$

Ostatecznie:

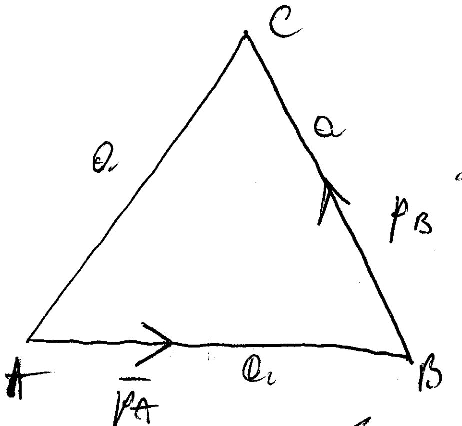
$$p_A = \omega_0^2 \frac{r_1+r_2}{r_2-r_1} \sqrt{\frac{r_1^2+r_2^2}{2}}$$

Zadanie 2

Dane: $\bar{p}_A \parallel \overline{AB}$, $\bar{p}_B \parallel \overline{BC}$

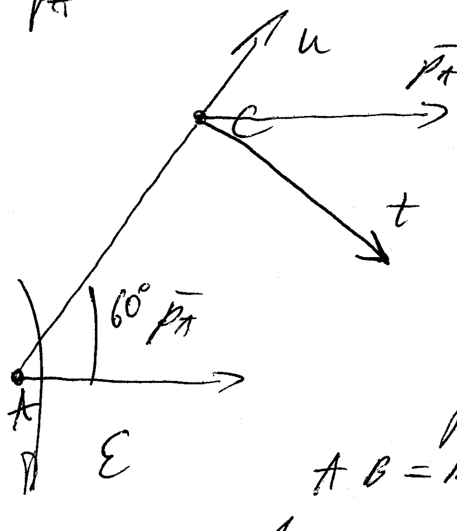
$p_A = p_B = p_0$

Szukamy: $\bar{p}_C$



Obliczamy $\bar{p}_C$ na podstawie $\bar{p}_A$

Zakładamy kierunek ε



$$p_{Cn} = p_A \cdot \cos\sphericalangle(\bar{p}_A, \bar{n}) - \omega^2 AC$$

$$p_{Ct} = p_A \cdot \cos\sphericalangle(\bar{p}_A, \bar{t}) + \varepsilon AC$$

$$p_{Cn} = p_0 \cdot \cos 60^\circ - \omega^2 AC$$

$$p_{Ct} = p_0 \cdot \cos 30^\circ + \varepsilon \cdot AC$$

$$AB = BC = CA = a$$

$$\left.\begin{aligned} p_{Cn} &= \tfrac{1}{2} p_0 - \omega^2 a \\ p_{Ct} &= \tfrac{\sqrt{3}}{2} p_0 + \varepsilon a \end{aligned}\right\}$$ nie znamy ω i ε, obliczamy podobnie jako (**)

Obliczamy teraz $\bar{p}_B$ na podstawie $\bar{p}_A$:

$$p_{Bn} = p_A \cdot \cos\sphericalangle(\bar{p}_A, \bar{n}) - \omega^2 AB$$
$$p_{Bt} = p_A \cos\sphericalangle(\bar{p}_A, \bar{t}) + \varepsilon AB$$
$$p_{Bn} = p_0 \cos 0^\circ - \omega^2 a$$
$$p_B = p_0 \cos 90^\circ + \varepsilon a$$

$$\left.\begin{aligned} p_{Bn} &= p_0 - \omega^2 a \\ p_{Bt} &= \varepsilon a \end{aligned}\right\} (*)$$

Znamy $\bar{p}_B$:

$$p_{Bn} = -p_B \cos 60^\circ$$
$$p_{Bt} = -p_B \sin 60^\circ$$

$$\left.\begin{aligned} p_{Bn} &= -p_0 \frac{1}{2} \\ p_{Bt} &= -p_0 \frac{\sqrt{3}}{2} \end{aligned}\right\}$$ wstawiamy do równania z (*)

$$\left.\begin{aligned} -\frac{1}{2} p_0 &= p_0 - \omega^2 a \\ -\frac{\sqrt{3}}{2} p_0 &= \varepsilon a \end{aligned}\right\}$$

$$\left.\begin{aligned} \omega^2 a &= \frac{3}{2} p_0 \\ \varepsilon a &= -\frac{\sqrt{3}}{2} p_0 \end{aligned}\right\}$$ wstawiamy do równania z (**)

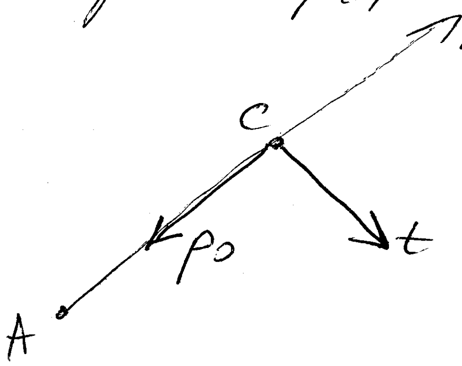
$$p_{Cn} = \frac{1}{2} p_0 - \frac{3}{2} p_0$$

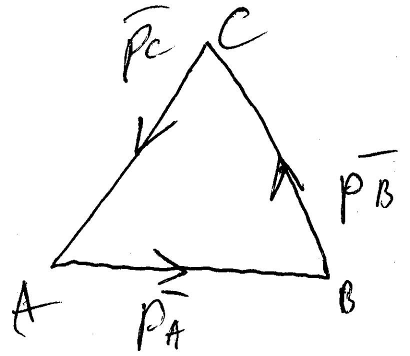
$$p_{Ct} = \frac{\sqrt{3}}{2} p_0 + \left(-\frac{\sqrt{3}}{2} p_0\right)$$

$$p_{Cn} = -p_0$$

$$p_{Ct} = 0$$

$$p_C = \sqrt{p_{Cn}^2 + p_{Ct}^2} = p_0$$





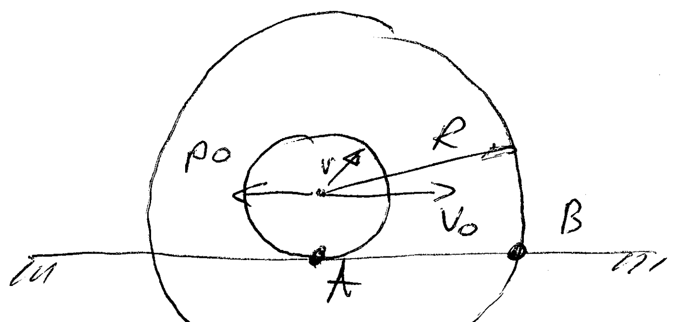
$\bar{p}_A \parallel \overline{AB}$

$\bar{p}_B \parallel \overline{BC}$ $\qquad p_A = p_B = p_C = p_0$

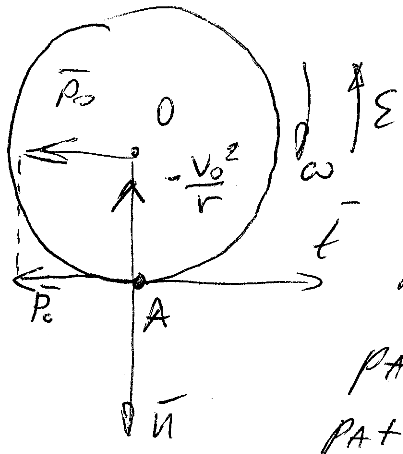
$\bar{p}_C \parallel \overline{CA}$

Zadanie

Koło o promieniu r z kołnierzem o promieniu R toczy się po nieruchomej szynie bez poślizgu z prędkością v_0 i przyspieszeniem p_0. Obliczyć przyspieszenie we wskazanych punktach A, B tego koła



Przyspieszenie p_A



$$p_{An} = p_0 \cos \sphericalangle(\bar{p}_0, \bar{n}) - \omega^2 OA$$
$$p_{At} = p_0 \cos \sphericalangle(\bar{p}_0, \bar{t}) + \varepsilon OA$$

$$p_{An} = p_0 \cos 90^\circ - \omega^2 r$$
$$p_{At} = p_0 \cos \sphericalangle 180^\circ + \varepsilon r$$

$$p_{An} = 0 - \omega^2 r$$
$$p_{At} = p_0(-1) + \varepsilon r$$

obliczam ω i ε

$$\omega = \frac{V_0}{r} \quad \varepsilon = \frac{p_0}{r}$$

$$p_{An} = -\left(\frac{V_0}{r}\right)^2 r = -\frac{V_0^2}{r}$$

$$p_{At} = -p_0 + \frac{p_0}{r} \cdot r = 0$$

$$p_{An} = -\frac{V_0^2}{r} \quad p_{At} = 0$$

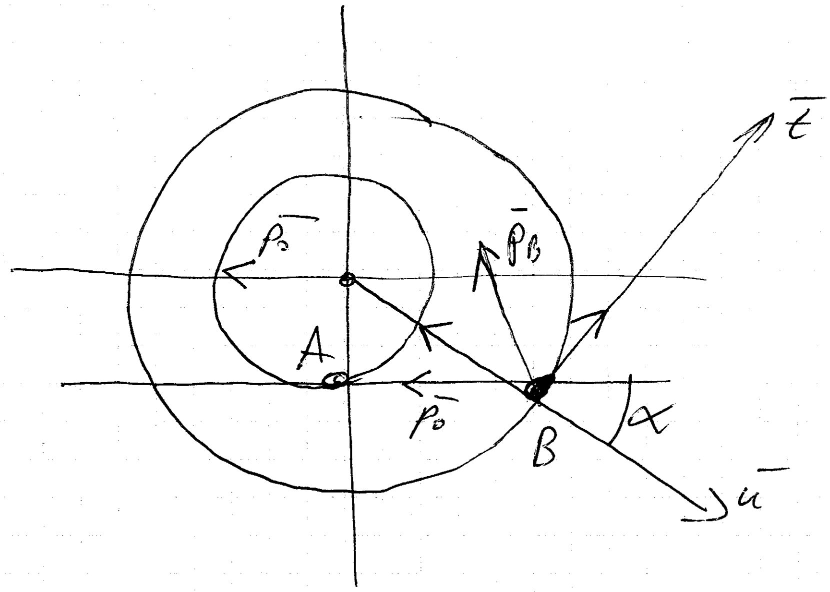
$$p_A = \sqrt{p_{An}^2 + p_{At}^2} = -\frac{V_0^2}{r}$$

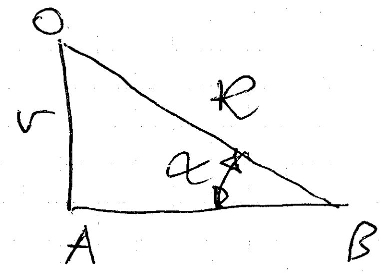
przyspieszenie p_B

$$p_{Bn} = p_0 \cos \sphericalangle(\bar{p}_0, \bar{n}) - \omega^2 OB$$
$$p_{Bt} = p_0 \cos \sphericalangle(\bar{p}_0, \bar{t}) + \varepsilon OB$$

$$\sphericalangle(\bar{p}_0, \bar{n}) = 180 - \alpha$$
$$\sphericalangle(\bar{p}_0, \bar{t}) = 90 + \alpha$$
$$OB = R$$

$$p_{Bn} = p_0 \cos(180^\circ - \alpha) - \omega^2 R$$
$$p_{Bt} = p_0 \cos(90^\circ + \alpha) + \varepsilon R$$
$$p_{Bn} = -p_0 \cos\alpha - \omega^2 R$$
$$p_{Bt} = -p_0 \sin\alpha + \varepsilon R$$





$$\sin\alpha = \frac{r}{R}$$

$$\cos\alpha = \frac{\sqrt{R^2-r^2}}{R}$$

$$p_{Bn} = -p_0\frac{\sqrt{R^2-r^2}}{R} - \frac{V_0^2}{r^2}R$$

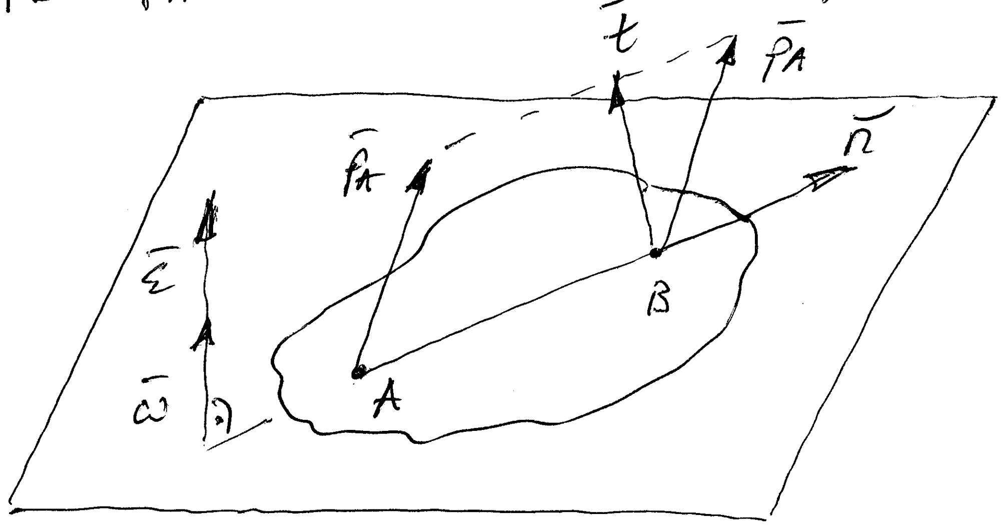
$$p_{Bt} = -p_0\frac{r}{R} + \frac{p_0}{r}R$$

$$p_B = \sqrt{p_{Bn}^2 + p_{Bt}^2}$$

$$p_B = \sqrt{\left(p_0\sqrt{1-\left(\frac{r}{R}\right)^2} + V_0^2\frac{R}{r}\right)^2 + p_0^2\left(\frac{R}{r}-\frac{r}{R}\right)^2}$$

Wyznaczanie przyspieszeń w ruchu płaskim

$$\bar{p}_B = \bar{p}_A + \bar{\varepsilon} \times \bar{\rho} + \bar{\omega} \times (\bar{\omega} \times \bar{\rho})$$

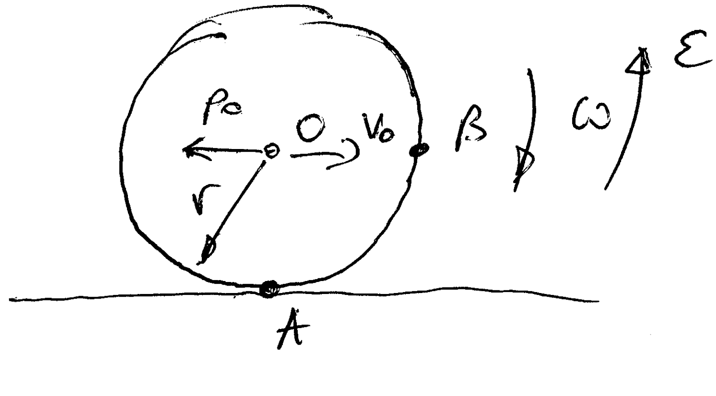


$$p_{Bn} = p_A \cdot \cos\sphericalangle(\bar{p}_A, \bar{n}) - \omega^2 AB$$

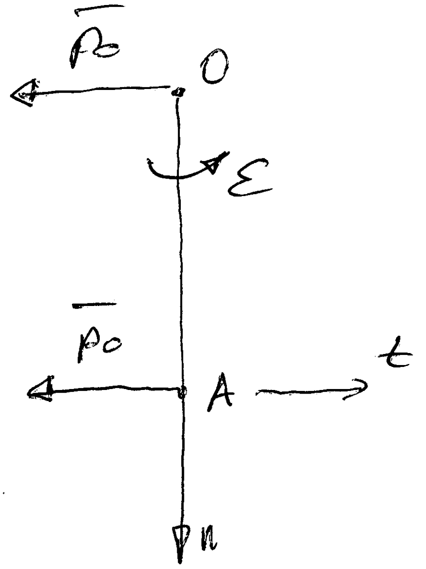
$$p_{Bt} = p_A \cdot \cos\sphericalangle(\bar{p}_A, \bar{t}) + \varepsilon \cdot AB$$

Zadanie 1

Znaleźć przyspieszenie $\bar{p}_A, \bar{p}_B$ wiedząc że dane jest r, V_0, p_0



Obliczamy przyspieszenie $\bar{p}_A$ na podstawie $\bar{p}_0$:



$$p_{An} = p_0 \cos\sphericalangle(\bar{p}_0, \bar{n}) - \omega OA$$
$$p_{At} = p_0 \cos\sphericalangle(\bar{p}_0, \bar{t}) + \varepsilon OA$$

$$p_{An} = p_0 \cos 90^\circ - \omega^2 r$$
$$p_{At} = p_0 \cos 180^\circ + \varepsilon r$$

$$p_{An} = p_0 \cdot 0 - \omega^2 r = -\omega^2 r$$
$$p_{At} = p_0 \cdot (-1) + \varepsilon r = -p_0 + \varepsilon r$$

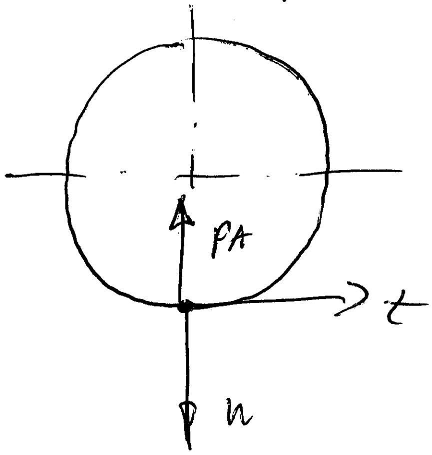
Dla koła toczącego się bez poślizgu

$$\left.\begin{array}{l} V_0 = \omega r \\ p_0 = \varepsilon r \end{array}\right\} \quad \begin{array}{l} \omega = \frac{V_0}{r} \\ \varepsilon = \frac{p_0}{r} \end{array}$$

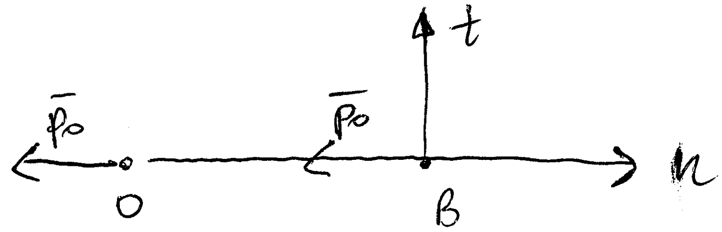
$$p_{An} = -\left(\frac{V_0}{r}\right)^2 \cdot r \quad \Rightarrow \quad p_{An} = -\frac{V_0^2}{r}$$

$$p_{At} = -p_0 + \frac{p_0}{\cancel{r}} \cdot \cancel{r} \quad \Rightarrow p_{At} = -p_0 + p_0 = 0$$

$$p_{An} = p_A = \frac{V_0^2}{r}$$



Obliczamy przyspieszenie $\bar{p}_B$ na podstawie $\bar{p}_0$



$p_{Bn} = p_0 \cos\sphericalangle(\bar{p}_0, \bar{n}) - \omega^2 OB$

$p_{Bt} = p_0 \cos\sphericalangle(\bar{p}_0, \bar{t}) + \varepsilon OB$

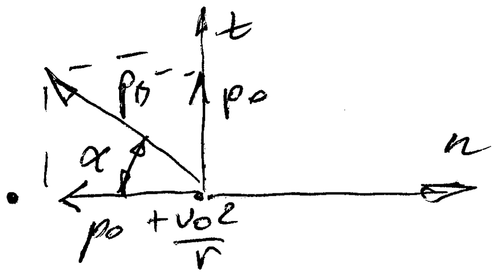
$p_{Bn} = p_0 \cdot \cos 180^\circ - \omega^2 r = -p_0 - \frac{V_0^2}{r}$

$p_{Bt} = p_0 \cos 90^\circ + \varepsilon r = \varepsilon r$

$p_{Bn} = -p_0 - \frac{V_0^2}{r} \Rightarrow p_{Bn} = -\left(p_0 + \frac{V_0^2}{r}\right)$

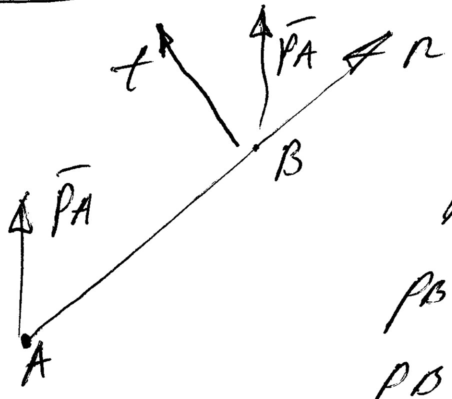
$p_{Bt} = p_0$

$p_B = \sqrt{p_{Bn}^2 + p_{Bt}^2} \Rightarrow p_B = \sqrt{\left(p_0 + \frac{V_0^2}{r}\right)^2 + p_0^2}$



$tg\alpha = \frac{p_0}{p_0 + \frac{V_0^2}{r}}$

Dodatkowo możemy obliczyć p_B na podstawie $\bar{p}_A$



$p_{Bn} = p_A \cdot \cos\sphericalangle(\bar{p}_A, \bar{n}) - \omega^2 AB$

$p_{Bt} = p_A \cdot \cos\sphericalangle(\bar{p}_A, \bar{t}) + \varepsilon AB$

$p_{Bn} = p_A \cdot \cos 45^\circ - \omega^2 AB$

$p_{Bt} = p_A \cdot \cos 45^\circ + \varepsilon AB$

$AB = \sqrt{2} \cdot r$

$p_{Bn} = \frac{V_0^2}{r} \cdot \frac{\sqrt{2}}{2} - \frac{V_0^2}{r^{\cancel{2}}} \cdot \sqrt{2}\,\cancel{r}$

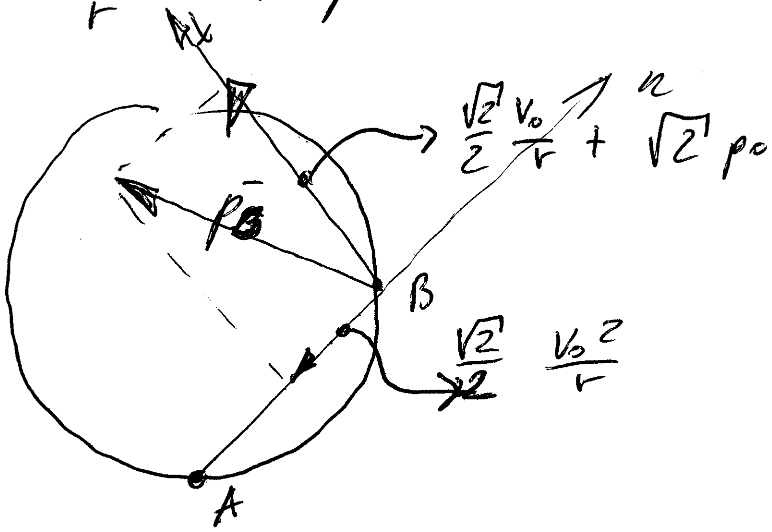
$p_{Bt} = \frac{V_0^2}{r} \cdot \frac{\sqrt{2}}{2} + \frac{p_0}{\cancel{r}} \cdot \sqrt{2}\,\cancel{r}$

$$p_{Bn} = \frac{v_0^2}{r}\left(\frac{\sqrt{2}}{2} - \sqrt{2}\right)$$

$$p_{Bt} = \frac{\sqrt{2}}{2} \cdot \frac{v_0^2}{r} + \sqrt{2} \cdot p_0$$

$$p_{Bn} = -\frac{\sqrt{2}}{2} \frac{v_0^2}{r}$$

$$p_{Bt} = \frac{\sqrt{2}}{2} \frac{v_0}{r} + \sqrt{2} p_0$$



$$|\bar{p}_A|^2 = p_A^2 = p_{An}^2 + p_{At}^2 = \left(-\omega_0^2 r_0 \frac{r_0}{r_s}\right)^2 +$$

$$+ \left(-\omega_0^2 r_0\right)^2 = \omega_0^4 r_0^2 \left(\frac{r_0^2}{r_s^2} + 1\right) =$$

$$= \omega_0^4 r_0^2 \frac{r_0^2 + r_s^2}{r_s^2} = \omega_0^4 \left(\frac{r_0}{r_s}\right)^2 \left(r_0^2 + r_s^2\right) =$$

$$= \omega_0^4 \left(\frac{r_1 + r_2}{r_2 - r_1}\right)^2 \left[\left(\frac{r_1 + r_2}{2}\right)^2 + \left(\frac{r_2 - r_1}{2}\right)^2\right] =$$

$$= \omega_0^4 \left(\frac{r_1 + r_2}{r_2 - r_1}\right)^2 \cdot \frac{1}{4}\left(r_1^2 + 2 r_1 r_2 + r_2^2 + r_2^2 -\right.$$

$$\left. - 2 r_1 r_2 + r_1^2\right) = \omega_0^4 \left(\frac{r_1 + r_2}{r_2 - r_1}\right)^2 \cdot \frac{2 r_1^2 + 2 r_2^2}{4} =$$

$$= \omega_0^4 \left(\frac{r_1 + r_2}{r_2 - r_1}\right)^2 \cdot \frac{r_1^2 + r_2^2}{2}$$

$$p_A = \omega_0^2 \frac{r_1 + r_2}{r_2 - r_1} \sqrt{\frac{r_1^2 + r_2^2}{2}}$$

Dla sprawdzenia, obliczamy moduł $\bar{p}_B$ w dwóch przypadkach dla przyspieszeń p_0 i p_A (które liczyliśmy)

$$p_B^2(O) = \left(p_0 + \frac{v_0^2}{r}\right)^2 + p_0^2$$

$$p_B^2(A) = \left(-\frac{\sqrt{2}}{2}\frac{v_0^2}{r}\right)^2 + \left(\frac{\sqrt{2}}{2}\frac{v_0^2}{r} + \sqrt{2}\,p_0\right)^2$$

$$p_B^2(O) = p_0^2 + 2p_0\frac{v_0^2}{r} + \frac{v_0^4}{r^2} + p_0^2$$

$$p_B^2(A) = \frac{v_0^2}{2r^2} + \frac{v_0^4}{2r^2} + 2\frac{\sqrt{2}}{2}\cdot\frac{v_0^2}{r}\sqrt{2}\,p_0 + 2p_0^2$$

$$p_B^2(O) = 2p_0^2 + 2p_0\frac{v_0^2}{r} + \frac{v_0^4}{r^2}$$

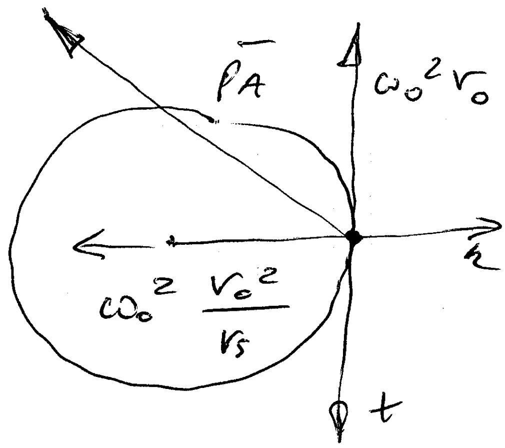
$$p_B^2(A) = \frac{v_0^4}{r^2} + 2\frac{v_0^2}{r}p_0 + 2p_0^2$$

$$p_{An} = -\omega_0^2 r_0 \frac{r_0}{r_s}$$

$$p_{At} = -\omega_0^2 r_0$$

$$tg\,\alpha = \frac{r_s}{r_0} \qquad r_0 = \frac{r_2 + r_1}{2}$$

$$tg\,\alpha = \frac{r_2 - r_1}{r_1 + r_2} \qquad r_s = \frac{r_2 - r_1}{2}$$



$$\frac{r_0}{r_s} = \frac{r_1 + r_2}{r_2 - r_1} > 1$$