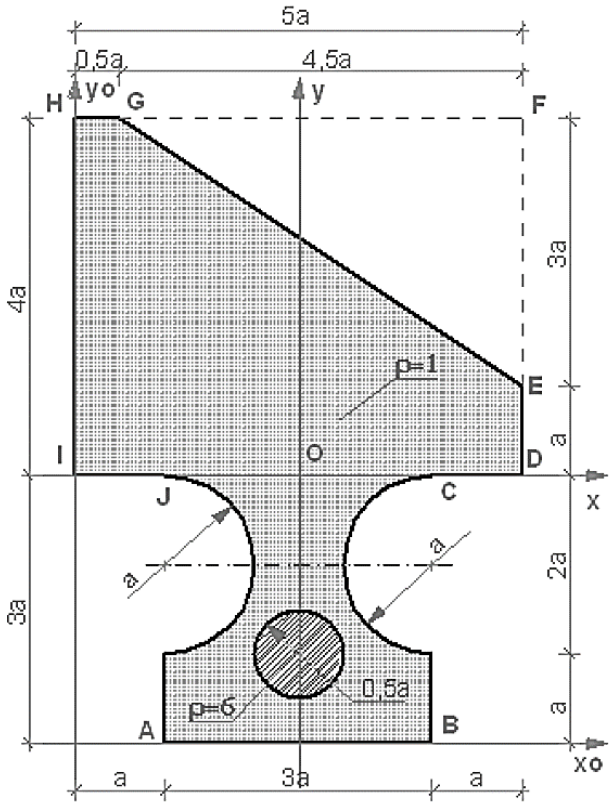
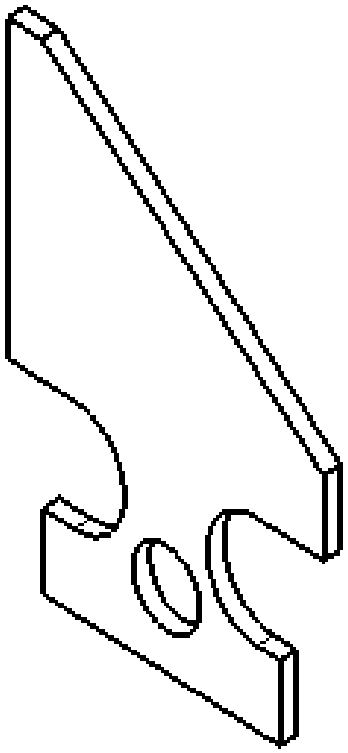
Zadanie

Wyznaczyć główne centralne osie i momenty bezwładności dla niejednorodnego przekroju o wymiarach jak na rysunku a=1m





Wyznaczenie środka ciężkoscu

Podział na obszary: $ABCD + DFHI - EFG - 2\cdot polkole + kolo\cdot\Delta\rho$

$$A(a) := (3\cdot a)^2 + 5\cdot a\cdot 4\cdot a - \frac{1}{2}\cdot 4.5\cdot a\cdot 3\cdot a - 2\cdot\frac{\pi\cdot a^2}{2} + \pi\cdot\left(\frac{a}{2}\right)^2\cdot 5 \qquad A(a) = 23.035\,m^2$$

$$S_x(a) := (3\cdot a)^2\cdot(-1.5a) + 5\cdot a\cdot 4\cdot a\cdot 2a - \left(\frac{1}{2}\cdot 4.5\cdot a\cdot 3\cdot a\cdot 3\right)a - 2\cdot\frac{\pi\cdot a^2}{2}\cdot(-a) + \pi\cdot\left(\frac{a}{2}\right)^2\cdot 5\cdot(-2a) \qquad S_x(a) = 1.538\,m^3$$

$$S_y(a) := -\left(\frac{1}{2}\cdot 4.5\cdot a\cdot 3\cdot a\right)\cdot a \qquad S_y(a) = -6.750\,m^3$$

$$x_c(a) := \frac{S_y(a)}{A(a)} \qquad y_c(a) := \frac{S_x(a)}{A(a)} \qquad x_c(a) = -0.293\,m$$

$$y_c(a) = 0.067\,m$$

Wyznaczenie momentów bezwładności w układzie Oxy

$$I_x(a) := \left[\frac{(3\cdot a)^4}{3} + \frac{5\cdot a\cdot(4\cdot a)^3}{3} - \left[\frac{4.5\cdot a\cdot(3\cdot a)^3}{36} + \left(\frac{1}{2}\cdot 4.5\cdot a\cdot 3\cdot a \right)\cdot(3\cdot a)^2 \right] \right] \ldots$$

$$+ -2\cdot\left(\frac{\pi\cdot a^4}{8} + \frac{\pi\cdot a^2}{2}\cdot a^2 \right) + \left[\frac{\pi\cdot\left(\frac{a}{2}\right)^4}{4}\cdot 5 + \pi\cdot\left(\frac{a}{2}\right)^2\cdot 5\cdot(2\cdot a)^2 \right] \qquad I_x(a) = 81.568\,\mathrm{m}^4$$

$$I_y(a) := \frac{(3\cdot a)^4}{12} + \frac{4\cdot a\cdot(5\cdot a)^3}{12} - \left[\frac{3\cdot a\cdot(4.5\cdot a)^3}{36} + \left(\frac{1}{2}\cdot 3\cdot a\cdot 4.5\cdot a \right)\cdot a^2 \right] \ldots$$

$$+ -2\cdot\left[\frac{\pi\cdot a^4}{8} - \frac{\pi\cdot a^2}{2}\cdot\left(\frac{4\cdot a}{3\cdot\pi} \right)^2 + \frac{\pi\cdot a^2}{2}\cdot\left[1.5\cdot a - \left(\frac{4\cdot a}{3\cdot\pi} \right) \right]^2 \right] + \frac{\pi\cdot\left(\frac{a}{2}\right)^4}{4}\cdot 5 \qquad I_y(a) = 30.464\,\mathrm{m}^4$$

$$I_{xy}(a) := 0 - \left[-\frac{(3a)^2\cdot(4.5a)^2}{72} + \left(\frac{1}{2}\cdot 3\cdot a\cdot 4.5\cdot a \right)\cdot(3a)\cdot a \right] \qquad I_{xy}(a) = -17.719\,\mathrm{m}^4$$

Główne momenty bezwładności w punkcie O

$$I_1(a) := \frac{I_x(a) + I_y(a)}{2} + \sqrt{\left(\frac{I_x(a) - I_y(a)}{2} \right)^2 + I_{xy}(a)^2} \qquad I_1(a) = 87.110\,\mathrm{m}^4$$

$$I_2(a) := \frac{I_x(a) + I_y(a)}{2} - \sqrt{\left(\frac{I_x(a) - I_y(a)}{2} \right)^2 + I_{xy}(a)^2} \qquad I_2(a) = 24.922\,\mathrm{m}^4$$

$$\alpha_1(a) := \operatorname{atan}\left(\frac{I_{xy}(a)}{I_y(a) - I_1(a)} \right)\cdot\frac{180}{\pi} \qquad \alpha_2(a) := \operatorname{atan}\left(\frac{I_{xy}(a)}{I_y(a) - I_2(a)} \right)\cdot\frac{180}{\pi}$$

$$\alpha_1(a) = 17.370 \qquad \alpha_2(a) = -72.630 \qquad |\alpha_1(a)| + |\alpha_2(a)| = 90.000$$

Centralne momenty bezwładności

$$I_{xc}(a) := I_x(a) - A(a)\cdot y_c(a)^2 \qquad I_{xc}(a) = 81.465\text{m}^4$$

$$I_{yc}(a) := I_y(a) - A(a)\cdot x_c(a)^2$$

$$I_{xcyc}(a) := I_{xy}(a) - A(a)\cdot x_c(a)\cdot y_c(a) \qquad I_{xcyc}(a) = -17.268\text{m}^4$$

Główne centralne momenty bezwładności

$$I_{1c}(a) := \frac{I_{xc}(a) + I_{yc}(a)}{2} + \sqrt{\left(\frac{I_{xc}(a) - I_{yc}(a)}{2}\right)^2 + I_{xcyc}(a)^2} \qquad I_{1c}(a) = 86.597\ \text{m}^4$$

$$I_{2c}(a) := \frac{I_{xc}(a) + I_{yc}(a)}{2} - \sqrt{\left(\frac{I_{xc}(a) - I_{yc}(a)}{2}\right)^2 + I_{xcyc}(a)^2} \qquad I_{2c}(a) = 23.355\ \text{m}^4$$

$$\alpha_{1c}(a) := \text{atan}\left(\frac{I_{xcyc}(a)}{I_{yc}(a) - I_{1c}(a)}\right)\cdot\frac{180}{\pi} \qquad \alpha_{2c}(a) := \text{atan}\left(\frac{I_{xcyc}(a)}{I_{yc}(a) - I_{2c}(a)}\right)\cdot\frac{180}{\pi}$$

$$\alpha_{1c}(a) = 16.550 \qquad \alpha_{2c}(a) = -73.450 \qquad |\alpha_{1c}(a)| + |\alpha_{2c}(a)| = 90$$