RACHUNEK WEKTOROWY

$a\,[1, 2, 3]$ $\quad \vec{a}\circ\vec{b} = -1+4+3 = 6$ $\quad \cos\alpha = \dfrac{6}{\sqrt{84}}$

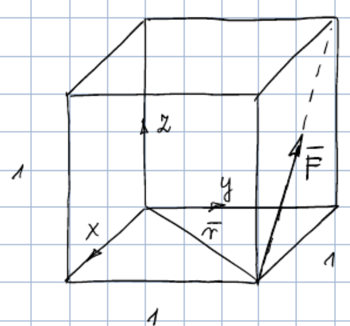
$b\,[-1, 2, 1]$ $\quad |a| = \sqrt{1+4+9} = \sqrt{14}$

$c\,[2, -3, 2]$ $\quad |b| = \sqrt{1+4+1} = \sqrt{6}$

$a \times b$ $\quad a\times b = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -1 & 2 & 1 \end{vmatrix} = -4\vec{i} - 4\vec{j} + 4\vec{k} = [-4, -4, 4]$

$a\circ(b\times c)$ $\quad b\times c = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 1 \\ 2 & -3 & 2 \end{vmatrix} = [7, 4, -1]$

$a\circ(b\times c) = 7+8-3 = 12$



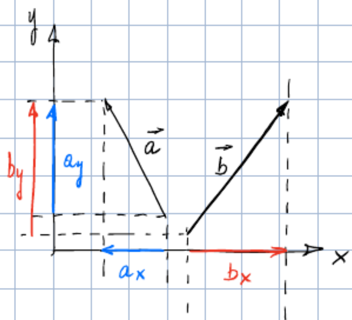
$\bar{M} = \bar{r}\times\bar{F}$

$\bar{r} = [1, 1, 0]$

$\bar{F} = \left[-F\cdot\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}F\right]$

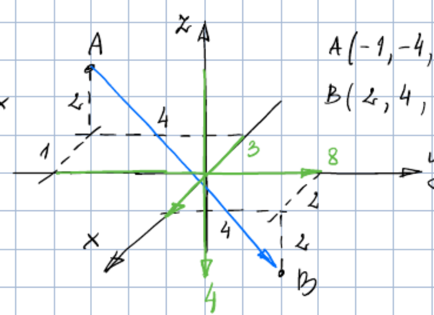
$\bar{M} = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ -F\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2}F \end{vmatrix} = \left[\frac{\sqrt{2}}{2}F, -\frac{\sqrt{2}}{2}F, \frac{\sqrt{2}}{2}F\right]$

RACHUNEK WEKTOROWY



sprowadzanie wektorów do współrzędnych

WEKTOR geometryczna interpretacja



$A(-1, -4, 2)$

$B(2, 4, 2)$

$$\overline{AB} = \begin{bmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ -4 \end{bmatrix}$$

Cztery wektory

$$\bar{a} = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix} \quad \bar{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \bar{c} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} \quad \bar{d} = \begin{bmatrix} -2 \\ -1 \\ -3 \end{bmatrix}$$

a) DODAWANIE

$$\bar{w} = \bar{a} + \bar{b} + \bar{c} + \bar{d} = \begin{bmatrix} a_x + b_x + c_x + d_x \\ a_y + b_y + c_y + d_y \\ a_z + b_z + c_z + d_z \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$$

b) ODEJMOWANIE

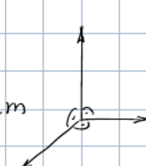
$$\bar{w} = \bar{a} - \bar{b} - \bar{c} = \begin{bmatrix} a_x - b_x - c_x \\ a_y - b_y - c_y \\ a_z - b_z - c_z \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

c) MNOŻENIE

przez SKALAR $\quad \bar{w} = -5 \cdot \bar{a} = \begin{bmatrix} -5 \cdot a_x \\ -5 \cdot a_y \\ -5 \cdot a_z \end{bmatrix} = \begin{bmatrix} -10 \\ 15 \\ 0 \end{bmatrix}$

SKALARNE służy do OPISANIA PRACY

$$s = \bar{b} \circ \bar{c} = \underbrace{b_x \cdot c_x + b_y \cdot c_y + b_z \cdot c_z}_{\longrightarrow \text{ w układzie prostokątnym}} = -4$$



w dowolnym układzie

$$s = \bar{b} \circ \bar{c} = |\bar{b}| \cdot |\bar{c}| \cdot \cos(\sphericalangle \bar{b}, \bar{c})$$

$\bar{b} \perp \bar{c} \Rightarrow \bar{b} \cdot \bar{c} = 0$

$$\cos(\sphericalangle \bar{b}, \bar{c}) = \frac{b_x c_x + b_y c_y + b_z c_z}{|\bar{b}| \cdot |\bar{c}|} \qquad |\bar{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2} \qquad |\bar{c}| = \sqrt{c_x^2 + c_y^2 + c_z^2}$$

MNOŻENIE WEKTOROWE

układ prawoskrętna

w układzie prostokątnym

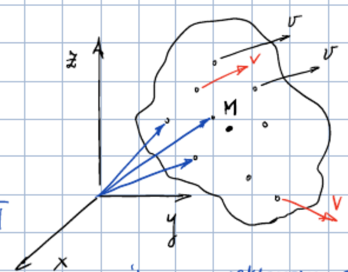
$$\bar{w} = \bar{b} \times \bar{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_x & b_y & b_z \\ d_x & d_y & d_z \end{vmatrix} = \vec{i} \begin{vmatrix} b_y & b_z \\ d_y & d_z \end{vmatrix} - \vec{j} \begin{vmatrix} b_x & b_z \\ d_x & d_z \end{vmatrix} + \vec{k} \begin{vmatrix} b_x & b_y \\ d_x & d_y \end{vmatrix} =$$

$$= \begin{bmatrix} b_y d_z - b_x d_y \\ b_z d_x - b_x d_z \\ b_x d_y - b_y d_x \end{bmatrix} = \begin{bmatrix} -6+1 \\ -6+3 \\ -3+4 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$$

W UKŁADZIE DOWOLNYM

$$|\bar{w}| = |\bar{b}| \cdot |\bar{d}| \cdot \sin(\sphericalangle b, d)$$

$\sqrt{w_x^2 + w_y^2 + w_z^2} = 0$ warunek równoległości $\bar{b}$ i $\bar{d}$



pęd

nie jest tak kiedy bryła porusza się bardziej skomplikowanym ruchem

iloczyn wektorowy sprowadza wektory do jednego punktu

MNOŻENIE MIESZANE

$(\bar{a} \circ \bar{b}) \times \bar{c}$ dlatego najpierw wykonujemy mnożenie wektorowe

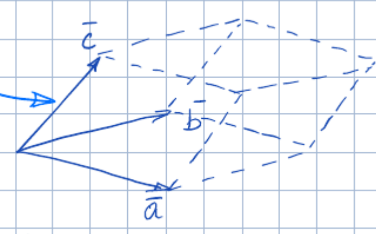
s nie można $s \times \bar{w}$

$$s = (\bar{a} \times \bar{b}) \circ \bar{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = a_x \begin{vmatrix} b_y & b_z \\ c_y & c_z \end{vmatrix} + a_y \begin{vmatrix} b_z & b_x \\ c_z & c_x \end{vmatrix} + a_z \begin{vmatrix} b_x & b_y \\ c_x & c_y \end{vmatrix}$$

$(\bar{a} \times \bar{b}) \circ \bar{c} = \bar{a} \circ (\bar{b} \times \bar{c})$

taki sam zapis niezależnie od tego jaki jest zapis

objętość



MNOŻENIE DWUWEKTOROWE

$$\bar{w} = \bar{a} \times \bar{b} \times \bar{c} = \bar{b} \cdot (\bar{a} \circ \bar{c}) - \bar{c} \cdot (\bar{a} \circ \bar{b})$$

ta postać jest równoważna

$$\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} (a_x c_x + a_y c_y + a_z c_z) - \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} (a_x b_x + a_y b_y + a_z b_z)$$

Zadanie 1

$$\bar{a} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \quad \bar{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \bar{c} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \bar{d} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \bar{e} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} \quad \bar{f} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$$\bar{w} = \bar{a} + \bar{b} - \bar{e} \times \bar{c} \circ \bar{d} \times \bar{c} \cdot (\bar{e} \times \bar{c} + \bar{f}) = \begin{bmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{bmatrix} - \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ e_x & e_y & e_z \\ c_x & c_y & c_z \end{vmatrix} \circ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ d_x & d_y & d_z \\ c_x & c_y & c_z \end{vmatrix} \cdot \left(\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ e_x & e_y & e_z \\ c_x & c_y & c_z \end{vmatrix} + \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \right)$$

Zadanie 2 SKALOWANIE

$$a = 5 \left[\frac{km}{s}\right] \qquad a = X \cdot \left[\frac{m}{h}\right]$$

$$a = 5 \cdot \frac{1000\ m}{\frac{1}{3600} h} = 5 \cdot 1000 \cdot 3600 \cdot \left[\frac{m}{h}\right]$$

$$a = 36 \left[\frac{\sqrt{kg}^{\,0,45}}{s^4 \sqrt[3]{N} \cdot KM} \right] \qquad a = X \cdot \left[\frac{\sqrt{g}^{\,0,45}}{h^4 \cdot \sqrt[3]{kG} \cdot W} \right] \qquad 1KM = 735,5W$$

$$a = 36 \cdot \left[\frac{\sqrt{(1000g)}^{\,0,45}}{\left(\frac{1}{3600} h\right)^4 \sqrt[3]{\frac{1}{9,81} kG} \cdot 735,5W} \right] = \underbrace{36 \cdot \frac{\sqrt{1000}^{\,0,45}}{\left(\frac{1}{3600}\right)^4 \cdot \sqrt[3]{\frac{1}{9,81}} \cdot 735,5}}_{\text{WSPÓŁCZYNNIK SKALUJĄCY}} \cdot \left[\frac{\sqrt{g}^{\,0,45}}{h^4 \sqrt[3]{kG}\, W} \right]$$